Structural Damage Identification using Low Frequency Non-Resonance Harmonic Excitation

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Abstract. The aim of this work is to present a damage identification method dedicated to beams. The core of the approach is the Virtual Distortion Method, which is a fast reanalysis method successfully applied to damage identification. Loss of stiffness and mass are modelled by virtual distortions and modifications of the parameters are calculated as a result of a sensitivity-based minimisation. In this paper we deal with a steady-state problem i.e. low frequency, non-resonance harmonic excitation induces a static-like structural response with virtual distortions (design variables) modelling parameter modifications.

Introduction

In this paper, the problem of structural damage identification in the frequency domain is investigated. The principal incentive for developing the new frequency-based approach was the reduction of vast consumption of computational time, observed in the previous time-domain approach [1]. A simplified dynamic problem with no damping is considered. A number of selected non-resonance excitation frequencies of low range (below 1 kHz) are the subject of analysis. Steady-state dynamic responses are provoked and a static-like inverse dynamic problem is posed in the framework of the Virtual Distortion Method [2], belonging to the class of model updating methods in Structural Health Monitoring. As a consequence, the gradient-based optimization process in the frequency domain turned out to be significantly faster compared to the one in the time domain, while the accuracy of identification remained unchanged. The proposed approach has been implemented in a software code [3].

The beam model has been chosen for presentation of the approach. Stiffness and mass reduction are considered as damage parameters. Experimental verification of the frequency-based approach is on the way for a 70-element space truss structure. An in-field demonstration is planned.

VDM with Harmonic Excitation

The use of VDM in dynamic damage identification was previously discussed in [4, 5, 6]. The papers dealt with modifications of stiffness parameters of truss and beams structures in the time domain, in which dynamic analysis using the VDM may be numerically time consuming. In this paper an alternative approach to damage identification in the frequency domain is discussed.

Virtual Distortions and Modification Parameters. The virtual distortion is an initial perturbation introduced to a finite element (or node) of an original structure subjected to external load (harmonic excitation in this paper). It may take the form of a strain $\varepsilon^0_\alpha(t,\omega)$ (applied as a pair of self-equilibrated forces in elements) – modelling stiffness modifications or a single unequilibrated force $p^0_k(t,\omega)$ (applied at nodes) – modelling inertia modification (t, ω denote time and frequency of excitation, respectively). Greek indices refer to elements whereas Latin ones to nodes.

Relation between virtual distortions $\varepsilon^0_\alpha(t,\omega)$, $p^0_k(t,\omega)$ and the parameter of modified stiffness $\mu_\alpha = \frac{\hat{k}^{EA}_\alpha}{k^{EA}_\alpha}$ called *modification parameter* is derived from the equivalence of internal forces and strains in the structure modelled by virtual distortions and the modified structure. It is expressed by the formula (see [4]):

$$\mu_\alpha\, \varepsilon_\alpha(t,\omega) = \varepsilon_\alpha(t,\omega) - \varepsilon^0_\alpha(t,\omega), \tag{1}$$

With Eq. (1), modifications of both the Young's modulus E as well as cross-section area A of an element α can be modelled. The updated strain $\varepsilon_\alpha(t,\omega)$ depends on virtual distortions $\varepsilon^0_\alpha(t,\omega)$ and $p^0_k(t,\omega)$, thus Eq. (1) is non-linear.

Any deformation state for a 2D-Beam finite element specifies 3 components (orthogonal base) obtained by solving the eigenvalue problem of its stiffness matrix. In this element, the virtual distortions corresponding to the 3 components are imposed. The virtual distortions have an oscillating form (presented in Fig. 1 for amplitude values) due to harmonic excitation of frequency ω. For 2D-Beam finite

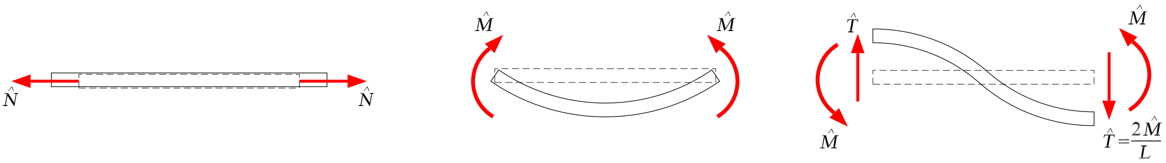

Figure 1: Basic virtual distortion states.

element, the relations analogous to Eq. (1) are expressed by the equations:

$$\mu^{(1)}_\alpha\, \varepsilon^{(e)}_\alpha(t,\omega) = \varepsilon^{(e)}_\alpha(t,\omega) - \varepsilon^{(e)\,0}_\alpha(t,\omega),$$
$$\mu^{(2)}_\alpha\, \kappa^{(e)}_\alpha(t,\omega) = \kappa^{(e)}_\alpha(t,\omega) - \kappa^{(e)\,0}_\alpha(t,\omega), \qquad \mu^{(3)}_\alpha\, \chi^{(e)}_\alpha(t,\omega) = \chi^{(e)}_\alpha(t,\omega) - \chi^{(e)\,0}_\alpha(t,\omega). \tag{2}$$

The first equation of the set (2) concerns axial stiffness (similarly to Eq. (1)) and the remaining ones describe bending states, where $\mu^{(2)}_\alpha = \mu^{(3)}_\alpha = \frac{\hat{k}^{EJ}_\alpha}{k^{EJ}_\alpha}$ is the ratio of a modified bending stiffness to the original one. Further, we assume the modifications of cross-section area ($\mu^{(1)}_\alpha = \frac{\hat{A}_\alpha}{A_\alpha}$) and moment of intertia ($\mu^{(2)}_\alpha = \mu^{(3)}_\alpha = \frac{\hat{J}_\alpha}{J_\alpha}$) independently.

For a steady-state harmonic excitation, the virtual distortions can be written in the following form:

$$\varepsilon^0_\alpha(t,\omega) = \varepsilon^0_\alpha(\omega)\, \sin(\omega\, t), \qquad p^0_k(t,\omega) = p^0_k(\omega)\, \sin(\omega\, t), \tag{3}$$

where $\varepsilon^0_\alpha(\omega)$ and $p^0_k(\omega)$ are amplitudes of the generated virtual distortions. In the next equations, the amplitudes are used assuming the following notation:

$$\varepsilon^0_\alpha(\omega) = \varepsilon^0_\alpha, \qquad p^0_k(\omega) = p^0_k, \tag{4}$$

where ω indicates the dependency on excitation frequency.

Let us introduce now the notion of *unit distortions*. Unit strain distortion $\varepsilon^0_\alpha(t,\omega)$ is an initial, oscillating strain imposed in finite element that would cause strain with unit amplitude (for $\omega = 0$ i.e. static case) in that element when taken out of structure. Unit virtual distortion $p^0_k(t,\omega)$ is an initial, oscillating force applied at node.

Influence Matrices. The crucial point for VDM calculations is the influence matrix containing amplitudes obtained for unit distortions. For steady-state problems two influence matrices are generated: influence matrix $B^{\varepsilon}_{i\beta}(\omega)$ storing displacements generated due to unit strain distortions $\varepsilon^0_{\beta}(\omega) = 1$ and influence matrix $B^{p}_{ik}(\omega)$ storing displacements generated due to unit force distortions $p^0_k(\omega) = 1$.

Knowing the virtual distortions ε^0_{α} and p^0_k and influence matrices, the updated response in displacements can be calculated (without re-computing the stiffness and mass matrices) as follows:

$$u_i = u^L_i + B^{\varepsilon}_{i\beta}\,\varepsilon^0_{\beta} + B^{p}_{ik}\,p^0_k, \tag{5}$$

where u^L_{α} denotes amplitudes of displacements of the original structure determined for the excitation frequency ω. Thus the actual response u_i depends on two virtual distortions ε^0_{α} and p^0_k. Multiplying Eq. (5) by $G_{\alpha Q}T_{Qi}$, the updated strain can be calculated as follows:

$$\varepsilon_{\alpha} = \varepsilon^L_{\alpha} + D^{\varepsilon}_{\alpha\beta}\,\varepsilon^0_{\beta} + D^{p}_{\alpha k}\,p^0_k, \tag{6}$$

where:

$$\varepsilon_{\alpha} = G_{\alpha Q}T_{Qi}\,u_i, \quad D^{\varepsilon}_{\alpha\beta} = G_{\alpha Q}T_{Qi}B^{\varepsilon}_{i\beta}, \quad D^{p}_{\alpha k} = G_{\alpha Q}T_{Qi}B^{p}_{ik}, \tag{7}$$

and $G_{\alpha Q}$ – geometric matrix, T_{Qi} – matrix of transformation to the global coordinate system.

It is necessary to quickly calculate the quantities q_A (e.g. displacement or strains), which correspond to the measured responses q^M_A. To this end, the generalized influence matrices $\breve{D}^{\varepsilon}_{A\alpha}$ and $\breve{D}^{p}_{Ak}$ are built utilizing the initial matrices: $B^{\varepsilon}_{i\beta}$, $D^{\varepsilon}_{\alpha\beta}$, B^{p}_{ik}, $D^{p}_{\alpha k}$. Finally, the updated response of a selected quantity (e.g strain) is determined in the following way:

$$q_A = q^L_A + \breve{D}^{\varepsilon}_{A\alpha}\,\varepsilon^0_{\alpha} + \breve{D}^{p}_{Ak}\,p^0_k, \tag{8}$$

where q^L_A denotes amplitudes of the requested responses of the original structure.

Problem Formulation

Generally, the equations of motion for a finite element model are expressed by the formula:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{9}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively and $\mathbf{f}(t)$ is the vector of external forces. Neglecting the influence of damping and expressing external load and displacement for a steady-state problem as:

$$\mathbf{f}(t) = \mathbf{f}\sin(\omega t), \qquad \mathbf{u}(t) = \mathbf{u}\sin(\omega t), \tag{10}$$

we can reshape Eq. (9) to read:

$$-\omega^2\mathbf{M}\mathbf{u} + \mathbf{T}\mathbf{S}\varepsilon = \mathbf{f}, \tag{11}$$

where $\mathbf{T}$ – the local-global transformation matrix, $\mathbf{S}$ – the matrix containing components of axial EA and bending EJ stiffness for finite elements of the original structure. Now, we can write the equations of motion for the structure modelled by virtual distortions ε^0_{α} and p^0_k (mass and stiffness matrices are intact) and the modified structure (mass and stiffness matrices are changed):

$$-\omega^2\mathbf{M}\mathbf{u} + \mathbf{T}\mathbf{S}(\varepsilon - \varepsilon^{\mathbf{0}}) = \mathbf{f} + p^0, \tag{12}$$

$$-\omega^2\hat{\mathbf{M}}\mathbf{u} + \mathbf{T}\hat{\mathbf{S}}\varepsilon = \mathbf{f}. \tag{13}$$

Let us note that the following relation holds:

$$\mathbf{TS}(\varepsilon - \varepsilon^{\mathbf{0}}) = \mathbf{T\hat{S}}\varepsilon, \tag{14}$$

thanks to the static VDM postulate of internal forces and strains equivalence between the modelled and modified structure. Subtracting Eq. (12) from Eq. (13) and taking into account Eq. (14), one obtains:

$$\omega^2 \mathbf{\hat{M}u} = \omega^2 \mathbf{Mu} + p^0. \tag{15}$$

Eq. (15) forms the dynamic postulate of inertia forces equivalence between the modified and modelled structure. The difference $\hat{M}_{ij} - M_{ij}$ for beam structures can be expressed in the following way:

$$\hat{M}_{ij} - M_{ij} = \Delta M_{ij} = (\mu_\gamma^A - 1)\overset{\mathrm{A}}{M}{}_{ij}^{\gamma} + (\mu_\gamma^J - 1)\overset{\mathrm{J}}{M}{}_{ij}^{\gamma}, \tag{16}$$

where $\mu_\gamma^A = \frac{\hat{A}_\gamma}{A_\gamma}$, $\mu_\gamma^J = \frac{\hat{J}_\gamma}{J_\gamma}$ are modification parameters for element γ. The mass matrix is decomposed into the matrix $\overset{\mathrm{A}}{M}{}_{ij}^{\gamma}$ – depending on the cross-section area A_γ and the matrix $\overset{\mathrm{J}}{M}{}_{ij}^{\gamma}$ – depending on the moment of intertia J_γ. Let us note that the following relation holds:

$$M_{ij} = \sum_\gamma \overset{\mathrm{A}}{M}{}_{ij}^{\gamma} + \sum_\gamma \overset{\mathrm{J}}{M}{}_{ij}^{\gamma}. \tag{17}$$

In order to determine the virtual distortions ε_α^0, p_k^0, let us substitute Eq. (6) to Eq. (1) and Eq. (5) to Eq. (15), yielding:

$$\begin{bmatrix} \delta_{\alpha\beta} - (1-\mu_\alpha)\, D_{\alpha\beta}^{\varepsilon} & -(1-\mu_\alpha)\, D_{\alpha k}^{p} \\ -\omega^2 \Delta M_{ij} B_{j\beta}^{\varepsilon} & \delta_{ik} - \omega^2 \Delta M_{ij} B_{jk}^{p} \end{bmatrix} \begin{bmatrix} \varepsilon_\beta^0 \\ p_k^0 \end{bmatrix} = \begin{bmatrix} (1-\mu_\alpha)\, \varepsilon_\alpha^L \\ \omega^2 \Delta M_{ij} u_j^L \end{bmatrix}. \tag{18}$$

The calculated virtual distortions ε_α^0, p_k^0 (for an assumed vector of stiffness parameters μ_α) from the set of equation (18) are used to compute the updated response q_A corresponding to the measured one q_A^M. Subsequently, the vector of stiffness modification parameters μ_α is iteratively determined by minimisation of the proposed objective function:

$$F(\mu_\alpha) = \sum_\omega \sum_A \left(q_A - q_A^M\right)^2, \tag{19}$$

using the following steepest descent optimisation approach:

$$\mu_\alpha^{(i+1)} = \mu_\alpha^{(i)} - \delta F^{(i)} \frac{\nabla F^{(i)}}{\nabla F^{(i)} \left[\nabla F^{(i)}\right]^T}, \tag{20}$$

where:

$$\nabla F^{(i)} = \frac{\partial F^{(i)}}{\partial \mu_\alpha^{(i)}} = \begin{bmatrix} \frac{\partial F^{(i)}}{\partial \varepsilon_\beta^{(i)0}} \frac{\partial \varepsilon_\beta^{(i)0}}{\partial \mu_\alpha^{(i)}} \\ \frac{\partial F^{(i)}}{\partial p_k^{(i)0}} \frac{\partial p_k^{(i)0}}{\partial \mu_\alpha^{(i)}} \end{bmatrix} = 2 \sum_\omega \sum_A \left(q_A - q_A^M\right) \begin{bmatrix} \breve{D}_{A\beta} \frac{\partial \varepsilon_\beta^{(i)0}}{\partial \mu_\alpha^{(i)}} \\ \breve{D}_{Ak} \frac{\partial p_k^{(i)0}}{\partial \mu_\alpha^{(i)}} \end{bmatrix}, \tag{21}$$

is the gradient of the objective function in $i-th$ iteration. For reaching the optimum solution of the function (19), the gradients $\frac{\partial \varepsilon_\beta^0}{\partial \mu_\alpha}$ and $\frac{\partial p_k^0}{\partial \mu_\alpha}$ have to be calculated. To this end, let us differentiate Eq. (18) with respect to modification parameters μ_α:

$$\begin{bmatrix} \delta_{\alpha\beta} - (1-\mu_\alpha)\, D_{\alpha\beta}^{\varepsilon} & -(1-\mu_\alpha)\, D_{\alpha k}^{p} \\ -\omega^2 \Delta M_{ij} B_{j\beta}^{\varepsilon} & \delta_{ik} - \omega^2 \Delta M_{ij} B_{jk}^{p} \end{bmatrix} \begin{bmatrix} \frac{\partial \varepsilon_\beta^0}{\partial \mu_\alpha} \\ \frac{\partial p_k^0}{\partial \mu_\alpha} \end{bmatrix} = \begin{bmatrix} -\varepsilon_\alpha \\ \omega^2 \frac{\partial \Delta M_{ij}}{\partial \mu_\alpha} u_j \end{bmatrix}. \tag{22}$$

Let us notice that the left-hand side matrix in Eq. (22) is the same as in Eq. (18), whereas the right-hand side depends now on updated displacements and strains.

Numerical Example

As an illustration of the discussed damage identification method, let us consider a simple cantilever beam divided into 25 finite elements, shown in Fig. 2. The original parameters are identical in all finite elements:

- cross-section area $A = 1 \cdot 10^{-4}\, m^2$,
- moment of intertia $J = 1.0417 \cdot 10^{-12}\, m^4$.

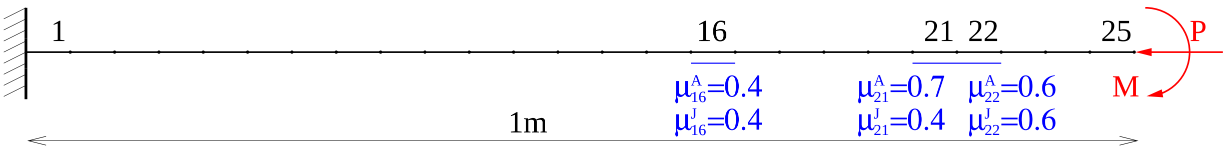

Figure 2: Tested 2D beam structure.

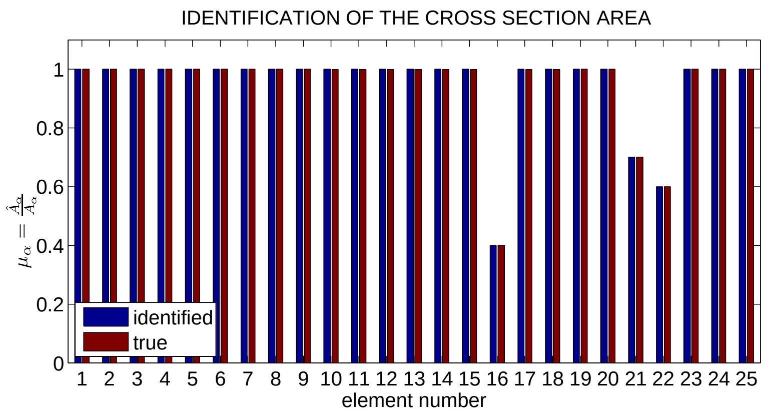

Figure 3: Identified cross-section areas after 500 iterations.

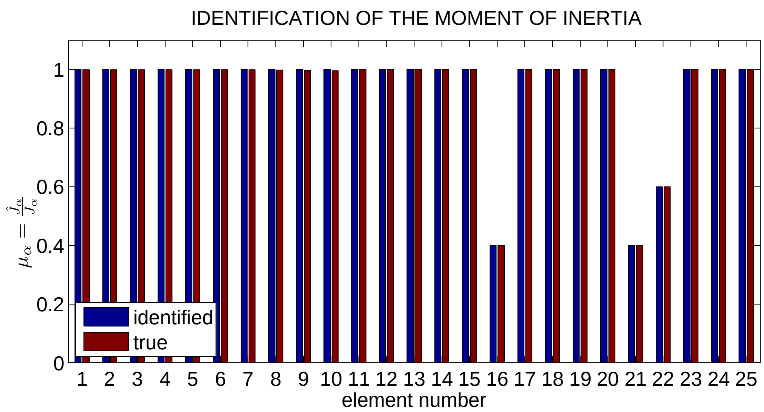

Figure 4: Identified moments of inertia after 500 iterations.

The harmonic excitation – bending moment $M = M^0 \sin(\omega t)$ and axial force $P = P^0 \sin(\omega t)$ – is applied to the free end of the cantilever beam. The amplitudes are: $M^0 = 1\,[Nm]$, $P^0 = 100\,[kN]$ and the arbitrary frequencies: $\omega = \{50, 100, 220\}\ [Hz]$ (out of resonance). The measured data were numerically simulated for each frequency ω (all components of strain responses ε^M_α). The results of inverse analysis are presented in Fig. 3 for the cross-section modification μ^A_α, and in Fig. 4 – for the moment of inertia modification μ^J_α.

Conclusions

Numerical effectiveness of the presented VDM-based approach in the frequency domain was demonstrated in the preceding section. Compared to the previous time domain approach, the computational effort has been reduced by 2 orders of magnitude. The experimental stand of a simply supported 3D steel truss structure (70 elements) is presented in Fig. 5b. The response of the structure is measured by thin piezo-patch sensors glued on elements (see Fig. 5a). There has to be lots of sensors in the truss, which is a consequence of using the frequency-domain, static-like approach. The experiment is now at the stage of matching model parameters to experimental responses. Various damage scenarios due to replacing the initial truss elements with other ones of different stiffness and mass will be investigated. An in-field demonstration for a steel railway bridge is envisaged in future research.

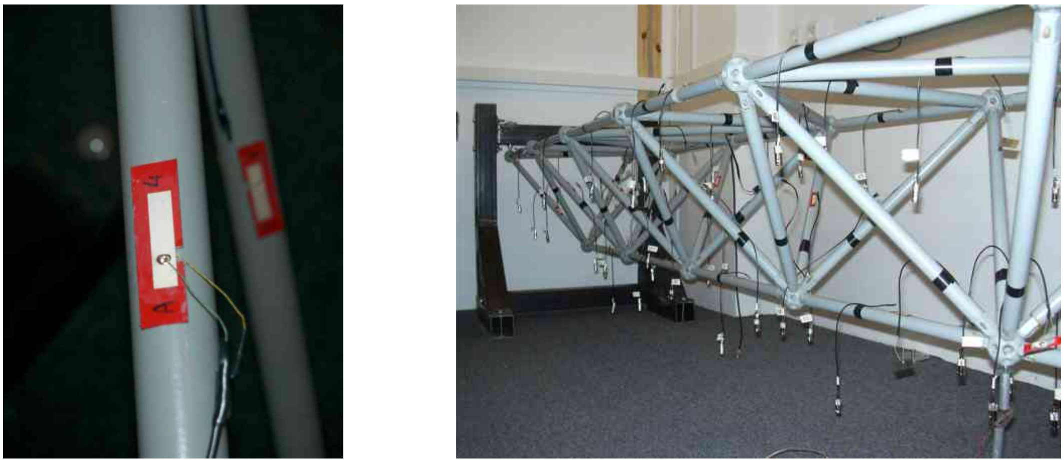

Figure 5: Experimental stand – 3D truss structure. (a) piezo-sensor, (b) general view.

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